Maths Grade 8 Knowledge Organiser

8.1 Manipulate and simplify surds

$\sqrt{25}$ is NOT a surd because it is exactly 5

$\sqrt{3}$ is a surd because the answer is not exact

A surd is an irrational number

- **To simplify surds look for square number factors**

$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

- Rules when working with surds:

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{3} \times \sqrt{15} = \sqrt{45} = \sqrt{9x5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

$m\sqrt{a} + n\sqrt{a} = (m+n)\sqrt{a}$

$2\sqrt{5} + 3\sqrt{5} = (2+3)\sqrt{5} = 5\sqrt{5}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt{\frac{72}{20}} = \frac{\sqrt{72}}{\sqrt{20}} = \frac{\sqrt{36} \times \sqrt{2}}{\sqrt{4} \times \sqrt{5}} = \frac{6\sqrt{2}}{2\sqrt{5}} = \frac{3\sqrt{2}}{\sqrt{5}}$

(Square number: $\sqrt{36}$; Square number: $\sqrt{4}$)

- **To rationalise the denominator**

This is the removing of a surd from the denominator of a fraction by multiplying both the numerator & the denominator by that surd

In general:

$\frac{a}{\sqrt{b}} =$

$= \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}}$ (Multiply both top & bottom by √b)

$= \frac{a\sqrt{b}}{b}$

Example

$\frac{6}{\sqrt{12}}$

$= \frac{6}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ (Multiply both top & bottom by √12)

$= \frac{6\sqrt{12}}{12} = \frac{\sqrt{12}}{2} = \frac{\sqrt{4} \times \sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

8.2 Upper & lower bounds

- **If 'a' is rounded to nearest 'x'**

Upper bound = a + $\frac{1}{2}$x

Lower bound = a – $\frac{1}{2}$x

e.g. if 1.8 is rounded to 1dp

Upper bound = 1.8 + $\frac{1}{2}$(0.1) = 1.85

Lower bound = 1.8 – $\frac{1}{2}$(0.1) = 1.75

- **Calculating using bounds**

Adding bounds

Maximum = Upper + upper

Minimum = Lower + lower

Subtracting bounds

Maximum = Upper - lower

Minimum = Lower – upper

Multiplying

Maximum = Upper x upper

Minimum = Lower x lower

Dividing

Maximum = Upper ÷ lower

Minimum = Lower ÷ upper

8.3 Solve quadratic equations by formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example

To solve: $3x^2 + 4x - 2 = 0$

a = 3

b = 4

c = -2

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$

$= \frac{-4 \pm \sqrt{16+24}}{6}$

$= \frac{-4 \pm \sqrt{40}}{6}$

$x = \frac{-4 + \sqrt{40}}{6}$ OR $\frac{-4 - \sqrt{40}}{6}$

x = 0.39(2dp) OR -1.72 (2dp)

8.4 Solve quadratic equation by completing the square

- **Make the coefficient of x^2 a square**

$2x^2 + 10x + 5 = 0$ (mult by 2)

⇨ $4x^2 + 20x + 10 = 0$

- **Add a number to both sides to make a perfect square**

$4x^2 + 20x + 10 = 0$ (Add 15)

$4x^2 + 20x + 25 = 15$

⇨ $(2x + 5)^2 = 15$

- **Square root both sides**

$2x + 5 = \pm\sqrt{15}$ (-5 from both sides)

$2x = -5 \pm \sqrt{15}$

$x = \frac{-5 + \sqrt{15}}{2}$ OR $\frac{-5 - \sqrt{15}}{2}$

$x = -0.56$ OR -4.44 (2dp)

8.5 Simplify algebraic fractions by factoring & camcelling

Example

$\frac{2x^2 + 3x + 1}{x^2 - 3x - 4}$ (factorise)

$= \frac{(2x + 1)(x + 1)}{(x - 4)(x + 1)}$ (cancel)

$= \frac{(2x + 1)}{(x - 4)}$

8.6 Transformation of functions

f(x) means 'a function of x'

e.g. $f(x) = x^2 - 4x + 1$

f(3) means work out the value of f(x) when x = 3

e.g. $f(3) = 3^2 - 4\times3 + 1 = -2$

In general for any graph y = f(x) these are the transformations

$y = f(x) + a$	Translation $\begin{pmatrix}0\\a\end{pmatrix}$
$y = f(x + a)$	Translation $\begin{pmatrix}-a\\0\end{pmatrix}$
$y = -f(x)$	Reflection in the x-axis
$y = f(-x)$	Reflection in the y-axis
$y = af(x)$	Stretch from the x-axis Parallel to the y-axis Scale factor=a
$y = f(ax)$	Stretch from the y-axis Parallel to the x-axis Scale factor= $\frac{1}{a}$

8.7 Similarity & enlargement

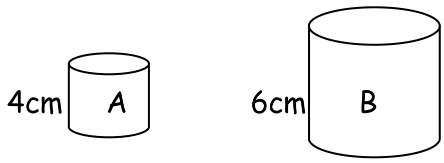
- **For similar shapes when:**

Length scale factor = **k**

Area scale factor = k^2

Volume scale factor = k^3

Example



If height of A = 4cm & height of B = 6cm

- Length scale factor = 6 ÷ 4 = **1.5**

If surface area of A = $132cm^2$

- Surface area of B = 132 x 1.5^2 = $297cm^3$

If volume of A = 120 cm^3

- Volume of B = 120 x 1.5^3 = $405cm^3$

8.8 Sine Rule (non-right angled triangles)

To find an angle use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

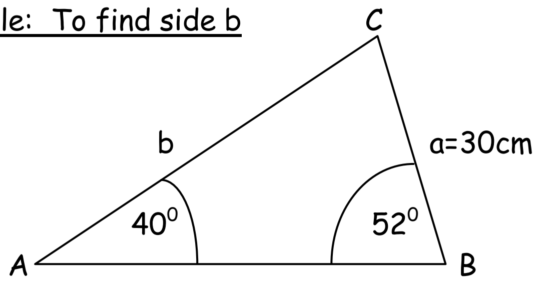
To find a side use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use SINE RULE when given:

- two sides and a non-included angle
- any two angles and one side

Example: To find side b



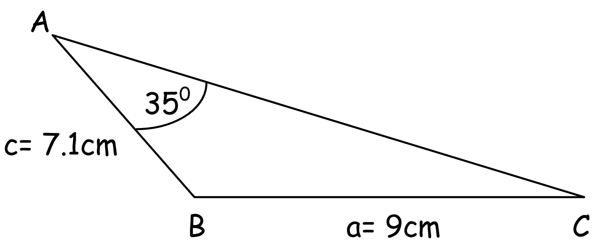
$$\frac{b}{\sin B} = \frac{a}{\sin C}$$

$$\frac{b}{\sin 52^0} = \frac{30}{\sin 40^0}$$

$$b = \frac{30}{\sin 40^0} \times \sin 52^0$$

b = 36.8 cm (1dp)

Example: To find angle C



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{7.1} = \frac{\sin 35^0}{9}$$

$$\sin C = \frac{\sin 35^0}{9} \times 7.1$$

$\sin C = 0.4524.....$

$C = \sin^{-1}(0.4524.....)$

$C = 28.9^0$(1dp)

8.8 Cosine Rule (non-right angled triangles)

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

OR

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

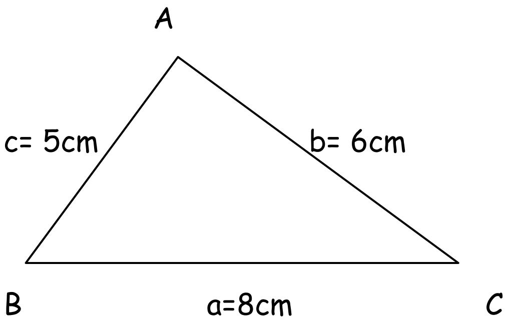
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Use COSINE RULE when given:

- 3 sides
- 2 sides and the included angle

Example: To find angle C



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

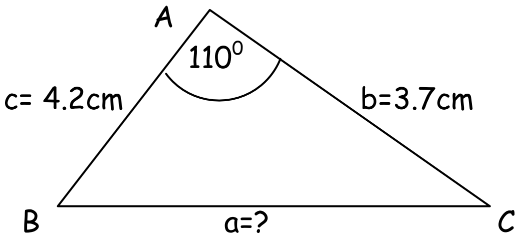
$$\cos C = \frac{8^2 + 6^2 - 5^2}{2\text{x}8\text{x}6}$$

$\cos C = 0.78125......$

$C = \cos^{-1}(0.78125......)$

$C = 38.6^0$(1dp)

Example: To find side a



$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 3.7^2 + 4.2^2 - 2\text{x}3.7\text{x}4.2 \cos 110^0$

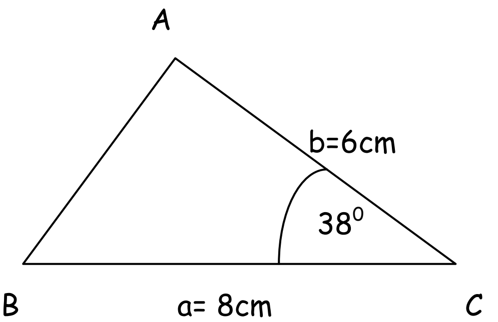
$a^2 = 41.96$

a = 6.48(2dp)

8.8 Area of triangle –height not known

$$\text{Area} = \tfrac{1}{2}\,ab\sin C$$
$$\text{Area} = \tfrac{1}{2}\,bc\sin A$$
$$\text{Area} = \tfrac{1}{2}\,ac\sin B$$

Example

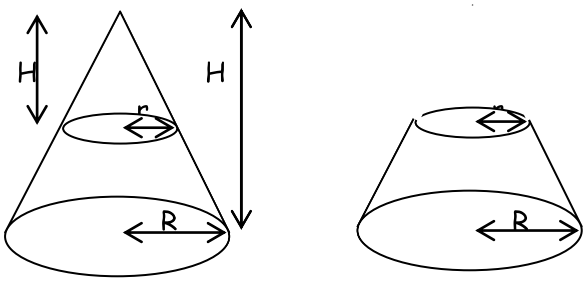


$\text{Area} = \tfrac{1}{2}\,ab\sin C$

$= \tfrac{1}{2} \times 8 \times 6 \times \sin 38^0$

$= 14.8\ cm^2$(1dp)

A*9 Frustum of a truncated cone



CONE FRUSTUM OF CONE

Volume of frustum

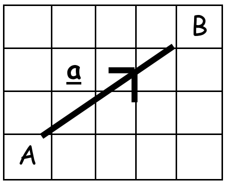
= Volume of whole cone – volume of cone removed

$= \frac{1}{3}\pi R^2H - \frac{1}{3}\pi r^2h$

8.10 Vectors

- **Vector notation**

This vector can be written as $\begin{pmatrix}3\\2\end{pmatrix}$ or $\underline{a}$ or $\overrightarrow{AB}$

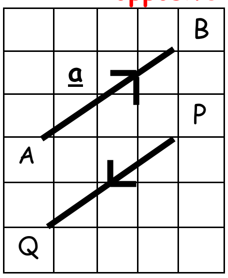


- **A vector has magnitude(length) & direction(shown by an arrow)**

Magnitude can be found by Pythagoras Theorem

$$AB = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.6$$

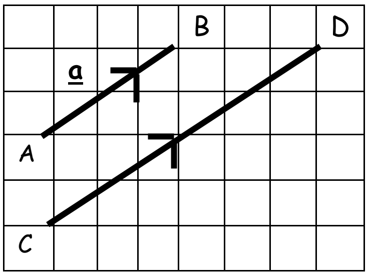
- **A parallel vector with same magnitude but opposite direction**



Vector $\overrightarrow{PQ}$ is equal in length to $\overrightarrow{AB}$ but opposite in direction so we say:

$$\overrightarrow{PQ} = -\underline{a}$$

- **A parallel vector with same direction but different magnitude**



Vector $\overrightarrow{CD}$ is twice (scalar 2) the magnitude but same direction so we say:

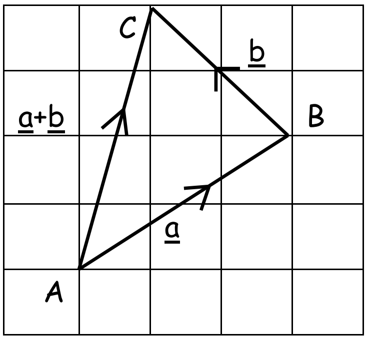
$$\overrightarrow{CD} = 2\underline{a}$$

A negative scalar would reverse the direction

- **Vector addition**

Adding graphically, the vectors go nose to tail

$\underline{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$



The combination of these two vectors:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \underline{a} + \underline{b}$$

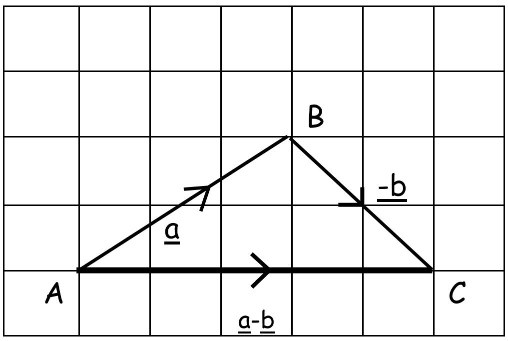
$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

- **Vector subtraction**

Adding graphically, the vectors go nose to tail

$\underline{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$



The combination of these two vectors:

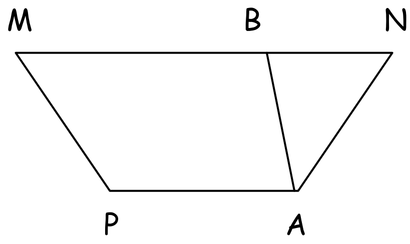
$$\overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AC} = \underline{a} - \underline{b}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$\overrightarrow{AC}$ is called the RESULTANT vector

- **The sum of vectors**



$$\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PM} + \overrightarrow{MB}$$

The vector AB is equal to the sum of these vectors or it could be a different route:

go via N

$$\overrightarrow{AB} = \overrightarrow{AN} + \overrightarrow{NB}$$

↑ start point ↑ end point

8.11 Probability – Tree diagram for successive dependent events

When events are dependent, the probability of the second event is called a conditional event because it is conditional on the outcome of the first event

Example

2 milk and 8 dark chocolates in a box
Kate chooses one and eats it. (ONLY 9 left now)
She chooses a second one
This can be shown on a tree diagram