Maths Grade 7 Knowledge Organiser

7/1 Use fractional & negative indices

- **Rules when working with indices:**

$a^x \times a^y = a^{(x+y)}$
$a^3 \times a^2 = a^{(3+2)} = a^5$
$2^3 \times 2^2 = 2^{(5)} = 32$

$a^x \div a^y = a^{(x-y)}$
$a^7 \div a^3 = a^{(7-3)} = a^4$
$3^7 \div 3^3 = 3^{(4)} = 81$

$(a^x)^y = a^{(x\,y)}$
$(a^3)^2 = a^6$
$(2^3)^2 = 2^6 = 64$

$a^0 = 1$
$y^0 = 1$
$8^0 = 1$

$a^{-x} = \frac{1}{a^x}$
$a^{-3} = \frac{1}{a^3}$
$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$a^{x/y} = (\sqrt[y]{a})^x$
$a^{2/5} = (\sqrt[5]{a})^2$
$32^{2/5} = (\sqrt[5]{32})^2 = 2^2$

$a^{-x/y} = \frac{1}{(\sqrt[y]{a})^x}$

7/2 Simplify surds

$\sqrt{25}$ is NOT a surd because it is exactly 5
$\sqrt{3}$ is a surd because the answer is not exact
A surd is an irrational number

- **To simplify surds look for square number factors**

$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

7/3 Upper & lower bounds

- **If 'a' is rounded to nearest 'x'**

Upper bound = a + $\frac{1}{2}$x
Lower bound = a - $\frac{1}{2}$x

e.g. if 1.8 is rounded to 1dp
Upper bound = 1.8 + $\frac{1}{2}$(0.1) = 1.85
Lower bound = 1.8 – $\frac{1}{2}$(0.1) = 1.75

- **Calculating using bounds**

Adding bounds
Maximum = Upper + upper
Minimum = Lower + lower

Subtracting bounds
Maximum = Upper - lower
Minimum = Lower – upper

Multiplying
Maximum = Upper x upper
Minimum = Lower x lower

Dividing
Maximum = Upper ÷ lower
Minimum = Lower ÷ upper

7/4 Direct and inverse proportion

The symbol $\propto$ means:
'varies as' or 'is proportional to'

- **Direct proportion**

If: $y \propto x$ or $y \propto x^2$ or $y \propto x^3$
Formulae: $y = kx$ or $y = kx^2$ or $y = kx^3$

Example

y is directly proportional to x
When y = 21, then x = 3
(find value of k first by substituting these values)

$y \propto x$ $\therefore y = kx$
21= k x 3
$\therefore$ k = 7
y = 7x

(Now this equation can be used to find y, given x)

- **Inverse proportion**

If: $y \propto \frac{1}{x}$ or $y \propto \frac{1}{x^2}$ or $y \propto \frac{1}{x^3}$

Formulae: $y = \frac{k}{x}$ or $y = \frac{k}{x^2}$ or $y = \frac{k}{x^3}$

Example

a is inversely proportional to b
When a = 12 and b = 4

$a \propto \frac{1}{b}$ $\therefore a = \frac{k}{b}$

$12 = \frac{k}{4}$

$\therefore k = 48$

$\therefore a = \frac{48}{b}$

7/5 Solve quadratic equation by factorising

- **Put equation in form $ax^2 + bx + c = 0$**

$2x^2 - 3x - 5=0$

- **Factorise the left hand side**

$(2x - 5)(x + 1) = 0$

- **Equate each factor to zero**

$2x - 5 = 0$ or $x + 1 = 0$

$x = 2.5$ or $x = -1$

7/6 Solve quadratic equations by formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

To solve: $x^2 + 4x - 2 = 0$

a = 1

b = 4

c = -2

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(3)}$

$= \frac{-4 \pm \sqrt{24}}{6}$

$= \frac{-4 \pm \sqrt{24}}{6}$

$x = \frac{-4 + \sqrt{24}}{6}$ OR $\frac{-4 - \sqrt{24}}{6}$

x = 0.45(2dp) OR -4.45 (2dp)

7/7 Simplify algebraic fractions

Adding & subtracting algebraic fractions

Example 1

$\frac{x + 3}{4} + \frac{x - 5}{3}$ (common denominator is 12)

$= \frac{3(x + 3)}{12} + \frac{4(x - 5)}{12}$

$= \frac{3x + 9 + 4x - 20}{12}$

$= \frac{7x - 11}{12}$

Example 2

$\frac{5}{(x + 1)} - \frac{3}{(x + 2)}$ (common denominator is (x+1)(x+2))

$= \frac{5(x + 2) - 3(x + 1)}{(x+1)(x+2)}$

$= \frac{5x + 10 - 3x - 3}{(x+1)(x+2)}$

$= \frac{2x + 7}{(x+1)(x+2)}$

7/8 Solve equations with fractions

$\frac{x}{2x - 3} + \frac{4}{x + 1} = 1$ Common denominator (2x-3)(x+1)

$\frac{x(x+1) + 4(2x-3)}{(2x-3)(x+1)} = 1$

$\frac{x^2 + x + 8x - 12}{(2x-3)(x+1)} = 1$

$x^2 + 9x - 12 = 1(2x-3)(x+1)$

$x^2 + 9x - 12 = 2x^2 - x - 3$ ($-x^2$ from both sides)

$9x - 12 = x^2 - x - 3$ (-9x from each side)

$-12 = x^2 - 10x - 3$ (+12 to each side)

$0 = x^2 - 10x + 9$ (factorise)

$(x + 9)(x + 1) = 0$

$x = -9$ or $x = -1$

7/9 Solve simultaneous equations ~ one is a quadratic

- Rewrite the linear with one letter in terms of the other
- Substitute the linear into the quadratic

Example

$x + y = 4$ (find one letter in terms of the other

⇨ $y = 4 - x$

$x^2 + y^2 = 40$ (substitute $y=4-x$)

$x^2 + (4-x)^2 = 40$ (Expand $(4-x)^2$)

$x^2 + 16 - 8x + x^2 = 40$

$2x^2 - 8x + 16 = 40$ (-40 from each side)

$2x^2 - 8x - 24 = 0$ (÷2 both sides)

$x^2 - 4x - 12 = 0$ (factorise)

$(x - 6)(x + 2) = 0$

$x = 6$ or $x = -2$

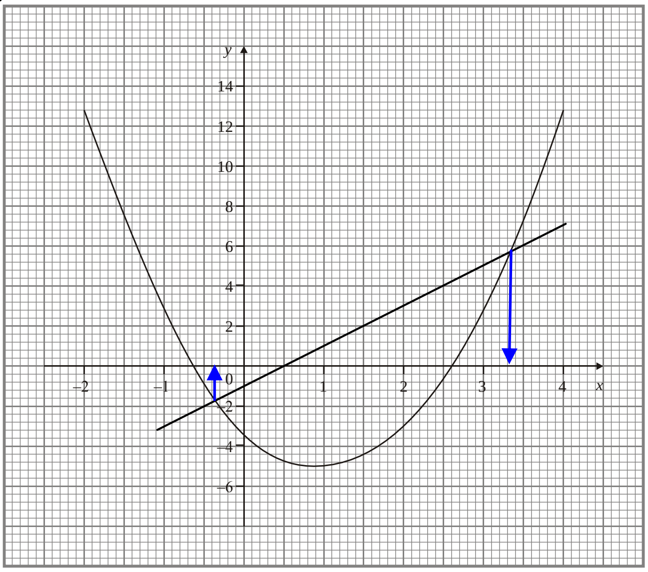
7/9 Solve GRAPHICALLY simultaneous equations ~ one is a quadratic

- Draw the two graphs and find where they intersect

Example

$y=2x^2-4x-3$

$y=2x-1$



Solutions are $x = -0.3$ and $x = 3.3$

(points of intersection)

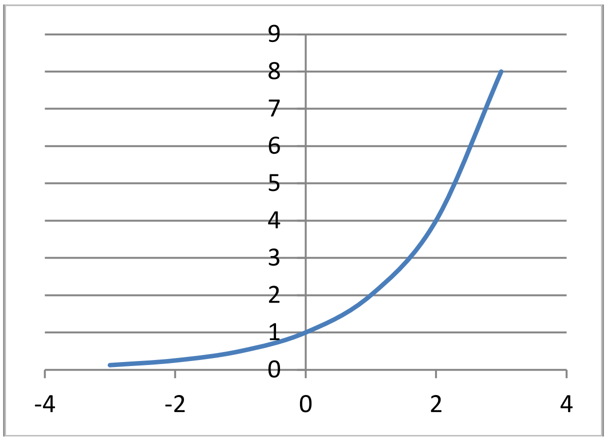
- Sometimes the equation has to be adapted~ rearrange the equation to solve so that the equation of the graph drawn is on the left. On the right is the other equation to be drawn

7/10 Graph of Exponential function

The graph of the exponential function is:

$$y = a^x$$

Example $y = 2^x$



It has no maximum or minimum point

It crosses the y-axis at (0,1)

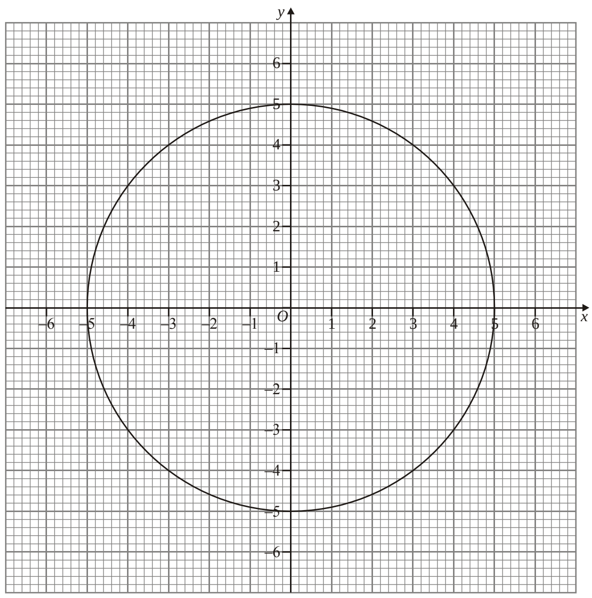
It never crosses the x-axis

7/11 Graph of the circle

The graph of a circle is of the form:

$$x^2 + y^2 = r^2$$

where r is the radius and the centre is (0,0)



This a circle of radius 5 and a centre (0,0)

The graph of this circle is

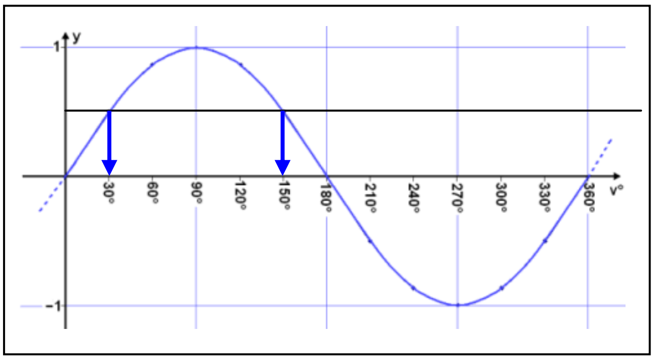
$$x^2 + y^2 = 5^2$$

⇨ $x^2 + y^2 = 25$

7/12 Graphs of trigonometric functions

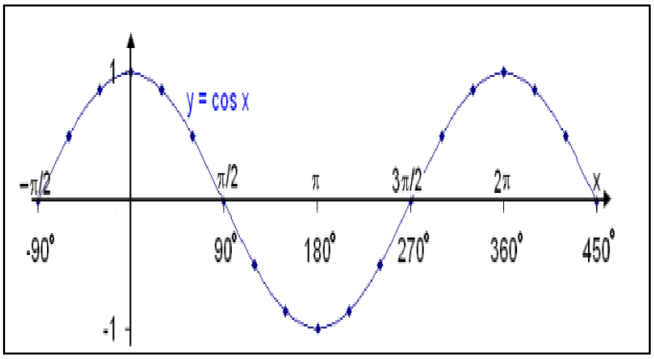
LEARN THE SHAPES OF THE GRAPHS

Graph of y=sin x



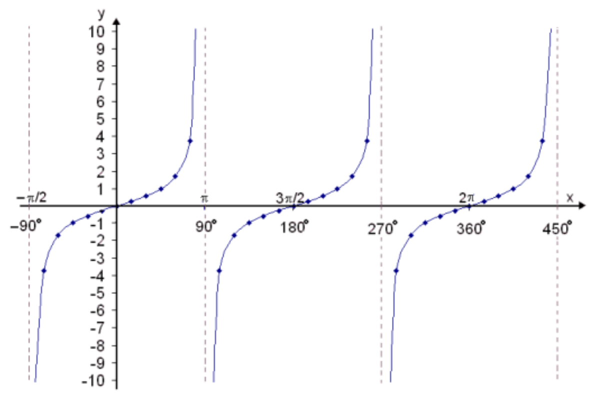
$-1 \leq \sin x \leq 1$

Graph y = cos x



$-1 \leq \cos x \leq 1$

Graph y = tan x



Tan x is undefined at 90^0, 270^0 ....

Solutions to trigonometrical equations can be found on the calculator and by using the symmetry of these graphs

Example:

If sin x = 0.5

x = 30^0 , 150^0 , *(See the solutions on sin graph above* or from calculator)

7/13 Change the subject of a formula

- **The subject may only appear once**

Use balancing to isolate the new subject

Example : To make 'x' the new subject

$A = \frac{k(x+5)}{3}$ (multiply both sides by 3)

⇨ $3A = k(x + 5)$ (Expand the bracket)

⇨ $3A = kx + 5k$ (-5k from both sides)

$3A - 5k = kx$ (÷ k both sides)

$\frac{3A - 5k}{k} = \frac{\cancel{k}x}{\cancel{k}}$

$x = \frac{3A - 5k}{k}$

- **The subject may appear twice**

Collect together all the terms containing the new subject & factorise to isolate it

Example: to make 'b' the new subject

$a = \frac{2 - 7b}{b - 5}$ (multiply both sides by (b – 5)

$a(b - 5) = 2 - 7b$ (Expand the bracket)

$ab - 5a = 2 - 7b$ (+7b to both sides)

$7b + ab - 5a = 2$ (+5a to both sides)
To leave terms in b together

$7b + ab = 2 + 5a$ (factorise the left side)
To isolate b

$\frac{b(7 + a)}{(7 + a)} = \frac{2 + 5a}{(7 + a)}$ (÷(7 + a) both sides)

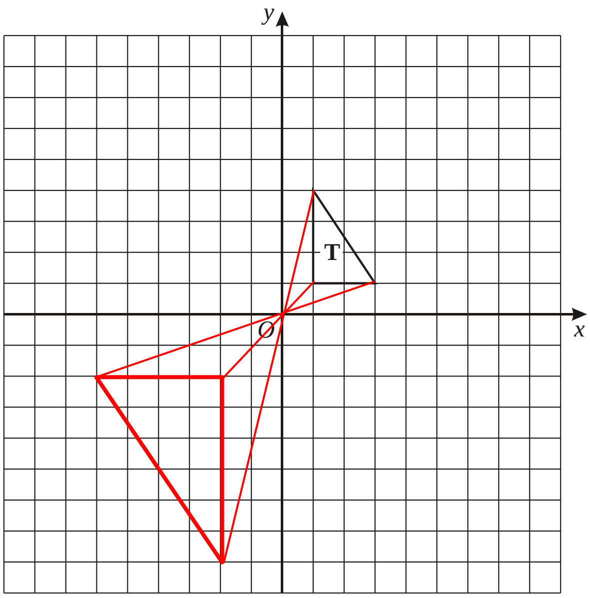
$b = \frac{2 + 5a}{(7 + a)}$

7/14 Enlarge by a negative scale factor

With a negative scale factor:

- The image is on the opposite side of the centre
- The image is also inverted

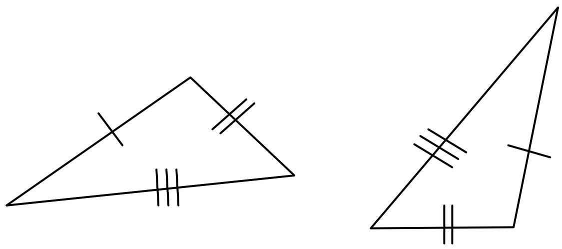
Example : Enlargement scale factor -2 about O



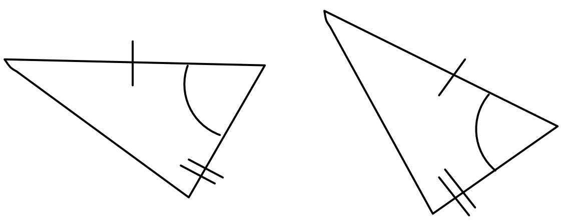
7/15 Congruence

- **Congruent shapes have the same size and shape, one will fit exactly over the other.**
- **2 triangles are congruent if any of these 4 conditions are satisfied on each triangle**

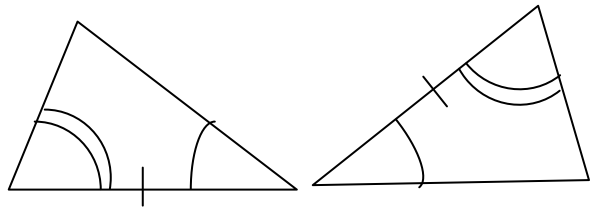
~The corresponding sides are equal ~ ***SSS***



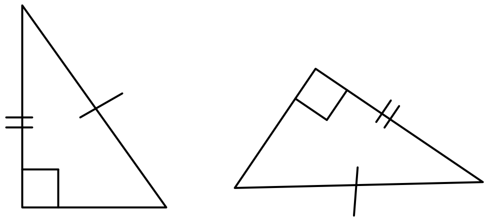
~2 sides & the included angle are equal ~ ***SAS***



~2 angles & the corresponding side are equal ~ ***ASA***



~Both triangles are right-angled, hypotenuses are equal and another pair of sides are equal ~ **RHS**



7/16 Similarity & enlargement

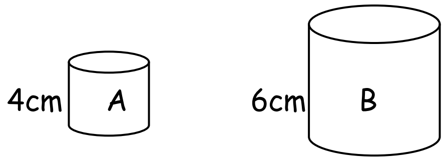
- **For similar shapes when:**

Length scale factor = **k**

Area scale factor = k^2

Volume scale factor = k^3

Example



If height of A = 4cm & height of B = 6cm

- Length scale factor = 6 ÷ 4 = **1.5**

If surface area of A = $132cm^2$

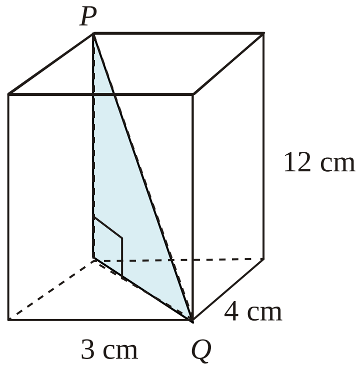
- Surface area of B = 132 x 1.5^2 = $297cm^3$

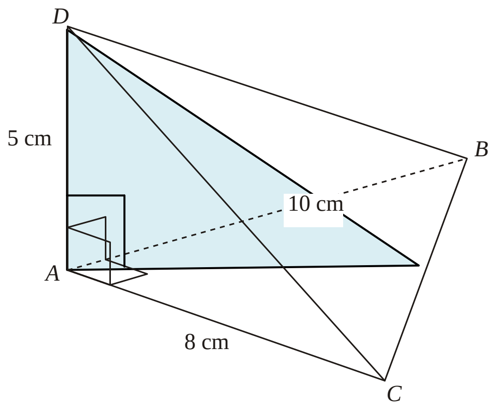
If volume of A = 120 cm^3

- Volume of B = 120 x 1.5^3 = $405cm^3$

7/17 Finding lengths & angles in 3D

- Identify the triangle in the 3D shape containing the unknown side/angle
- Use Pythagoras and trigonometry as appropriate



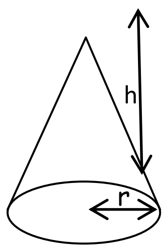


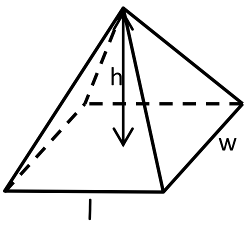
7/18 Pyramid & Sphere –Volume

VOLUME – PYRAMID

Volume of Pyramid = $\frac{1}{3}$ x area of cross-section x height

e.g. cone



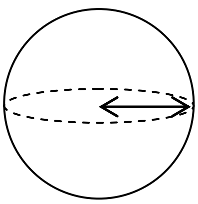


Volume = $\frac{1}{3} \times \pi r^2 h$

Volume = $\frac{1}{3} \times l \times w \times h$

VOLUME – SPHERE

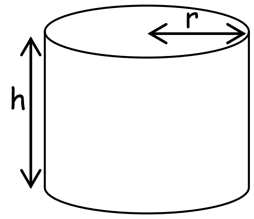
Volume of Sphere = $\frac{4}{3}\pi r^3$



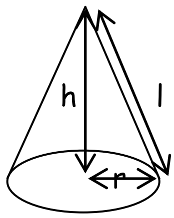
7/18 Pyramid & Sphere – Surface Area

CURVED SURFACE AREA

~Curved surface area of a cylinder = $2\pi rh$

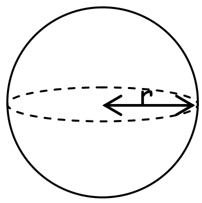


~Curved surface of a cone = πrl

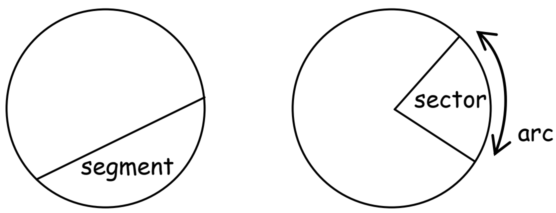


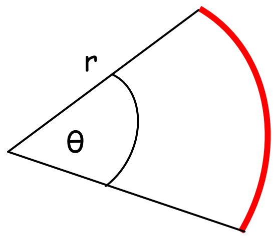
[NB To find 'l' use Pythagoras' Theorem
$l^2 = h^2 + r^2$]

~Curved surface of a sphere = $4\pi r^2$

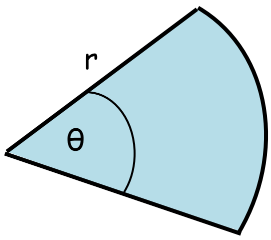


7/19 Length of arc & area of sector





Length of arc = $\frac{\theta}{360^0} \times 2\pi r$



Area of sector = $\frac{\theta}{360^0} \times \pi r^2$

7/23 Probability – the 'and' 'or' rule

P(A or B) = p(A) + p(B)

Use this addition rule to find the probability of either of two mutually exclusive events occurring

e.g. p(a 3 on a dice **or** a 4 on a dice)

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

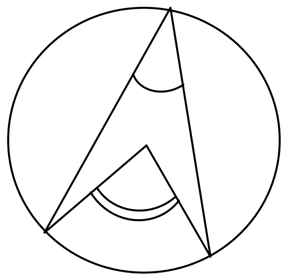
P(A and B) = p(A) x p(B)

Use this multiplication rule to find the probability of either of both of two independent events occurring

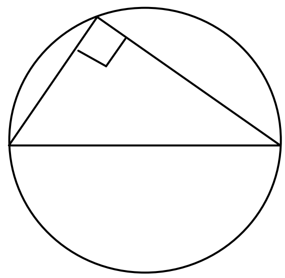
e.g. p(Head on a coin **and** a 6 on a dice)

$= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

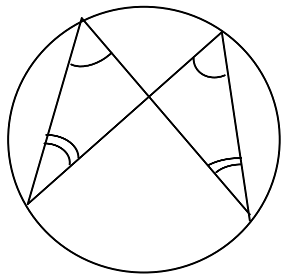
7/20 Circle properties



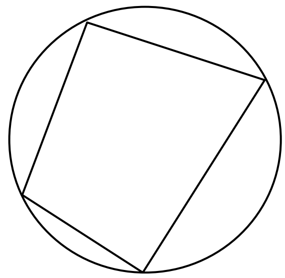
The angle at the centre = 2 x the angle at the circumference



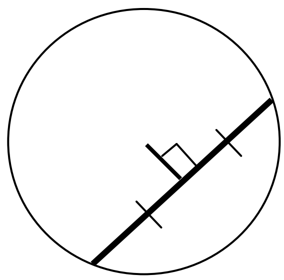
The angle in a semi-circle is a right angle



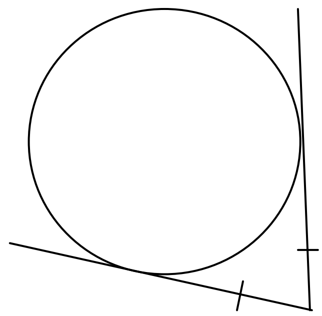
Angles in the same segment are equal



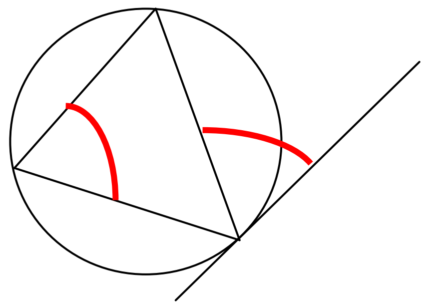
Opposite angles of a cyclic quadrilateral add up to 180^0



The perpendicular from the centre to a chord bisects the chord



Tangents from a point to a circle are equal



The angle between a tangent and a chord is equal to the angle in the alternate segment

7/21 Sampling

The sample is:

- a small group of the population.
- an adequate size
- representative of the population

Simple random sampling
Everyone has an equal chance
e.g. pick out names from a hat

Systematic sampling
Arranged in some sort of order
e.g. pick out every 10th one on the list

Stratified sampling
Sample is divided into groups according to criteria
These groups are called strata
A simple random sample is taken from each group in proportion to its size using this formula:

No from each group = $\frac{\text{Stratum size}}{\text{Population}} \times$ Sample size

Example

An inspector wants to look at the work of a stratified sample of 70 of these students.

Language	Number of students
Greek	145
Spanish	121
German	198
French	186
Total	**650**

No. from Greek = $\frac{145}{650} \times 70 \approx 16$

No. from Spanish = $\frac{121}{650} \times 70 \approx 13$

No. from German = $\frac{198}{650} \times 70 \approx 21$

No. from French = $\frac{186}{650} \times 70 \approx 20$

This only tells us 'how many' to take - now take a random sample of this many from each language

7/22 Histograms

- Class intervals are not equal
- Vertical axis is the frequency density
- The area of each bar not the height is the frequency

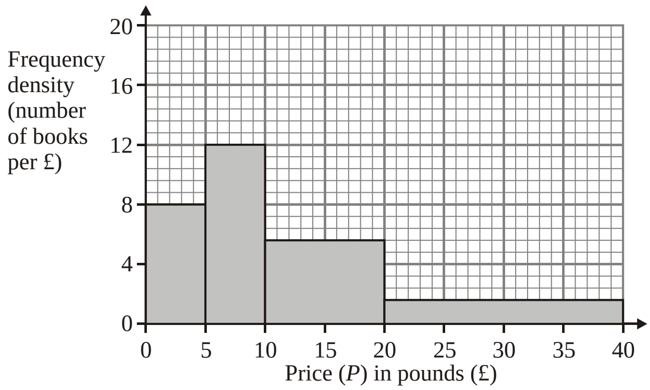
Frequency = class width x frequency density
Frequency density = frequency ÷ class width

To draw a histogram
Calculate the frequency density
Example

Age (x years)	Class width	f	Frequency density
$0 < x \le 20$	20	28	28÷20 = 1.4
$20 < x \le 35$	15	36	36÷15 = 2.4
$35 < x \le 45$	10	20	20÷10 = 2
$45 < x \le 65$	20	30	30÷20 = 1.5

Scale the frequency density axis up to 2.4
Draw in the bars to relevant heights & widths

To interpret a histogram



NOTE: On the vertical axis each small square = 0.8

Price (P) in pounds (£)	f = width x height
$0 < P \le 5$	5 x 8 = 40
$5 < P \le 10$	5 x 12 = 60
$10 < P \le 20$	10 x 5.6 = 56
$20 < P \le 40$	20 x 1.6 = 32